Welcome to Deep Institute

ISS COACHING

Learn with DEEP Institute

Dear Students,

This Institute is dedicated to cater the needs of students preparing for Indian Statistical Service. We publish videos on Youtube channel for student help.

Guided by - Sudhir Sir 9999001310

Sudhirdse1@gmail.com

www.isscoaching.com

2513, Basement, Hudson Lane Beside HDFC Bank Opp. Laxmi Dairy, GTB Nagar New Delhi: 110009

TEST SERIES PAPER-1 (TEST-1) SOLUTION

Q-1 $\Rightarrow$ $P(X=-2) = P(X=-1) = p_1$

$P(X=2) = P(X=1) = p_2$

$P(X=0) = p$

Since $P[X>0] = P[X<0] = P[X=0]$

$\Rightarrow \quad 2p_2 = 2p_1 = p$

Since total prob^t is 1 so

$2p_1 + 2p_2 + p = 1 \Rightarrow 3p = 1 \Rightarrow p = \frac{1}{3}$

$\Rightarrow p_2 = p_3 = \frac{1}{6}$

$$\Rightarrow \begin{array}{cccccc} X: & -2 & -1 & 0 & 1 & 2 \\ P(x): & \frac{1}{6} & \frac{1}{6} & \frac{2}{6} & \frac{1}{6} & \frac{1}{6} \end{array}$$

$\Rightarrow E(X) = \frac{1}{6}[-2-1+0+1+2] = 0$

$E(X^2) = \frac{1}{6}[4+1+0+1+4] = \frac{10}{6}$

$\Rightarrow V(X) = \frac{10}{6} - 0^2 = \frac{5}{3}$

$\Rightarrow E(X) + V(X) = \frac{5}{3} \Rightarrow$ b-option

Q-2 $\Rightarrow$ first we calculate c as

$c\int_0^\infty e^{-x} x^{4-1} dx = 1 \Rightarrow c \cdot \Gamma 4 = 1 \Rightarrow c \cdot 3! = 1 \Rightarrow c = \frac{1}{6}$

$\Rightarrow E\left(\frac{1}{x}\right) = \frac{1}{6}\int_0^\infty e^{-x} x^{3-1} dx = \frac{1}{6} \cdot \Gamma 3 = \frac{2}{6} = \frac{1}{3}$

$\Rightarrow$ c-option

Q-3 ⇒ first we find K as

$$K\int_0^\infty\int_0^\infty e^{-x-y}\,dx\,dy = 1 \quad \Rightarrow \quad K = 1$$

Now $P[X+Y<1] = \int_{x=0}^{1}\int_{y=0}^{1-x} e^{-x}\, e^{-y}\, dy\, dx$

$$= \int_{x=0}^{1} e^{-x}\left[\frac{e^{-y}}{-1}\right]_0^{1-x} dx = \int_{x=0}^{1} e^{-x}\left[1 - e^{-1+x}\right] dx$$

$$= \int_{x=0}^{1} \left(e^{-x} - e^{-1}\right) dx = \left[\frac{e^{-x}}{-1} - e^{-1}x\right]_0^1 = 1 - \frac{2}{e}$$

⇒ d-option

Q-4 ⇒ $X \sim \exp(1) \Rightarrow f(x) = e^{-x}$.

$Y = 3X + 5 \Rightarrow \frac{dY}{dX} = 3$

$\Rightarrow f_X(x) = 3\cdot f_Y(y). \Rightarrow \frac{1}{3}\cdot e^{-x} = f_Y(y)$

$\Rightarrow f_Y(y) = \frac{1}{3}\cdot e^{-\left(\frac{y-5}{3}\right)} \; ; \quad 5 \le y < \infty$

Since density f^n of Y is decreasing so mode is at $Y = 5$.

⇒ c-option

Q-5 ⇒ $P[X=n] \propto \frac{1}{3^n} \quad \forall \; n = 1, 2, 3, \cdots$

$\Rightarrow P[X=n] = \frac{K}{3^n}$

$\Rightarrow K \sum_{n=1}^{\infty} \frac{1}{3^n} = 1 \Rightarrow K\left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\right] = 1$

$\Rightarrow K\left[\frac{1/3}{1-1/3}\right] = 1 \Rightarrow K = 2$

$\Rightarrow P[X = n] = \frac{2}{3^n}$

$\Rightarrow E(x) = \sum_{n=1}^{\infty} n \cdot \frac{2}{3^n} = 2\left[\frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \cdots\right]$

let $S = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \frac{4}{3^4} + \cdots$

$\Rightarrow \frac{S}{3} = \frac{1}{3^2} + \frac{2}{3^3} + \frac{3}{3^4} + \cdots$

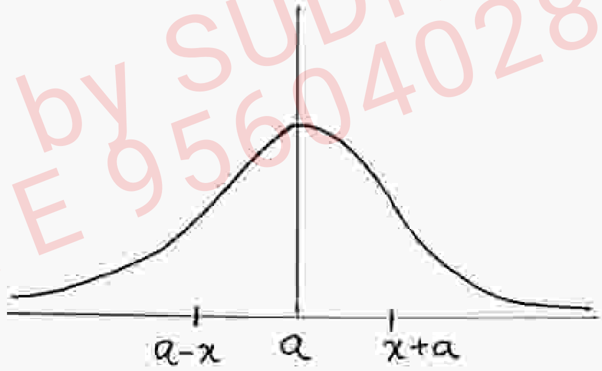
$\Rightarrow \left(1 - \frac{1}{3}\right) S = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots = \frac{1/3}{1-1/3} = \frac{1}{2}$

$\Rightarrow S = 3/4$

$\Rightarrow E(x) = 3/2 \Rightarrow$ c- option

Q-6 $\Rightarrow$

If density f^n of X is Symmetrical about the point $x = a$



then $f_x(a-x) = f_x(a+x)$ and

$F(a-x) = 1 - F(a+x)$

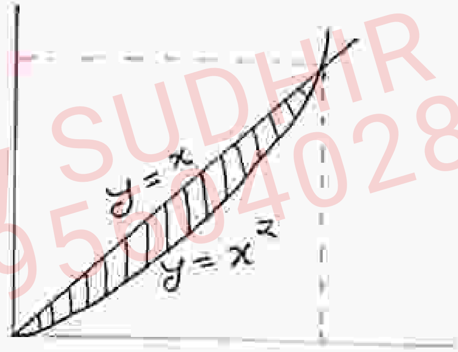
and also $E(x) = a$ if it Exists.

$\Rightarrow$ a- option

Q-7 ⇒ $f(x,y) = \begin{cases} 6 & ; \quad x^2 < y < x \quad ; \quad 0 < x < 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$

If $0 < x < 1 \Rightarrow x^2 \le y \le x$

If $0 < y < 1 \Rightarrow y \le x \le \sqrt{y}$



first we find $f_Y(y)$ as

$$f_Y(y) = \int_{x=y}^{\sqrt{y}} 6 \cdot dx = 6(\sqrt{y} - y) \quad ; \quad 0 < y < 1.$$

Now

$$f(x/y) = \frac{1}{\sqrt{y} - y} \quad ; \quad y < x < \sqrt{y} \quad ; \quad 0 < y < 1.$$

$$\Rightarrow E[X/Y = y] = \int_{x=y}^{\sqrt{y}} x \cdot \frac{1}{\sqrt{y} - y} dx = \frac{1}{\sqrt{y} - y}\left[\frac{x^2}{2}\right]_y^{\sqrt{y}} = \frac{\sqrt{y} + y}{2}$$

⇒ a-option

Q-8 ⇒ $P[\mu - K\sigma \le X \le \mu + K\sigma] \geq 0.95$

$\Rightarrow P[|X - \mu| \le K \cdot \sigma] \geq 0.95 \quad -(i)$

by chebyshev's Inequality

$$P[|X - \mu| \le K \cdot \sigma] \geq 1 - \frac{1}{K^2} \quad -(ii)$$

by Eq^n (i) and (ii)

$1 - \frac{1}{K^2} = 0.95 \Rightarrow K = 2\sqrt{5} \Rightarrow$ c-option.

Q-9 $\Rightarrow$ $P[X_n = \pm n^{-\frac{1}{2}}] = \frac{1}{2}$

$\Rightarrow E(X_k) = \frac{1}{2}\cdot\frac{1}{\sqrt{k}} + \frac{1}{2}\cdot\left(-\frac{1}{\sqrt{k}}\right) = 0$

$\Rightarrow E(X_k^2) = \frac{1}{2}\cdot\frac{1}{k} + \frac{1}{2}\cdot\frac{1}{k} = \frac{1}{k}$

$\Rightarrow V(X_k) = \frac{1}{k}$

$\Rightarrow B_n = V\left(\sum_{k=1}^{n} X_k\right) = \sum_{k=1}^{n} V(X_k) = \left[1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\right] < n$

$\Rightarrow \lim_{n\to\infty} \frac{B_n}{n^2} = 0$

$\Rightarrow$ W.L.L.N Holds $\Rightarrow$ b-option

Q-10 $\Rightarrow$ $X \sim b(n,p) \Rightarrow p(x) = {}^nC_x \cdot p^x q^{n-x}$; $q = 1-p$

let $Y = \frac{X}{n} \Rightarrow Y : 0, \frac{1}{n}, \frac{2}{n}, \ldots \frac{n}{n}$.

$\Rightarrow F_Y(y) = P[Y \le y] = P[X \le ny] = p(1) + p(2) + \cdots + p(ny)$

and $F_Y(y-1) = p(1) + p(2) + \cdots + p(n(y-1))$

$\Rightarrow P[Y=y] = F_Y(y) - F_Y(y-1) = p(ny) = {}^nC_{ny}\cdot p^{ny}\cdot q^{n-ny}$

$\Rightarrow$ a-option

Q-11 $\Rightarrow$ $M_X(t) = (q+pe^t)^n$

$M_Y(t) = (p+qe^t)^n$

$\Rightarrow M_{X+Y}(t) = [(q+pe^t)\cdot(p+qe^t)]^n$

$\Rightarrow M_{X+Y}(t) = [\, p\cdot q + (p^2+q^2)e^t + p\cdot q\, e^{2t}]^n$ — *

Now first we calculate M.G.F of X_i as

$M_{X_i}(t) = E[e^{tX_i}] = p(0)\cdot e^{t\cdot 0} + p(1)\cdot e^{t\cdot 1} + p(2)\cdot e^{t\cdot 2}$

$= p\cdot q + (p^2+q^2)e^t + p\cdot q\, e^{2t}$

$\Rightarrow M_{\Sigma X_i}(t) = [\, p\cdot q + (p^2+q^2)e^t + p\, q\, e^{2t}]^n$ — **

by Eq^n * and ** we can say that

$X+Y$ and $\sum_{i=1}^{n} X_i$ have same distribution

$\Rightarrow$ b-option

Q-12 $\Rightarrow f(x,y) = f_X(x)\cdot f_{Y/X}(y/x) = \dfrac{e^{-\lambda}\lambda^x}{x!}\cdot {}^{x}c_y \cdot p^y (1-p)^{x-y}$

$x = 0, 1, 2, 3, \ldots$

$y = 0, 1, 2, \ldots x$

$\Rightarrow P_Y(y) = \sum_{x=y}^{\infty} \dfrac{e^{-\lambda}\lambda^x}{x!}\cdot {}^{x}c_y \cdot p^y (1-p)^{x-y}$

$= e^{-\lambda} \sum_{x=y}^{\infty} \dfrac{\lambda^x}{x!}\cdot \dfrac{x!}{y!(x-y)!}\cdot p^y (1-p)^{x-y}$

$= \dfrac{e^{-\lambda} p^y}{y!}\cdot \lambda^y \sum_{x=y}^{\infty} \dfrac{\lambda^{x-y}(1-p)^{x-y}}{(x-y)!}$

$= \dfrac{e^{-\lambda}(\lambda p)^y}{y!}\cdot \sum_{x-y=0}^{\infty} \dfrac{\lambda^{x-y}(1-p)^{x-y}}{(x-y)!} = \dfrac{e^{-\lambda}(\lambda p)^y}{y!}\, e^{\lambda q}$

$= \dfrac{e^{-\lambda p}(\lambda p)^y}{y!} \Rightarrow Y \sim P(\lambda p) \Rightarrow$ b-option.

Q-13 ⇒ $f(x/y) = \dfrac{e^{-y}\, y^{x}}{x!}$; $x = 0, 1, 2, \ldots$

and $f(y) = e^{-y}$; $y \geq 0$.

$$\Rightarrow f(x, y) = \frac{e^{-2y} \cdot y^{x}}{x!}$$

Now for mode of X, we calculate $f_X(x)$ as

$$f_X(x) = \int_{y=0}^{\infty} \frac{e^{-2y}\, y^{x}}{x!}\, dy = \frac{1}{x!} \int_{y=0}^{\infty} e^{-2y} \cdot y^{(x+1)-1}\, dy$$

$$\Rightarrow f_X(x) = \frac{1}{x!} \frac{\sqrt{x+1}}{2^{x+1}} = \frac{1}{2^{x+1}} = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{x} \quad ; \; x = 0, 1, 2, \ldots$$

$$\Rightarrow X \sim g\left(\tfrac{1}{2}\right) \Rightarrow \text{mode} = 0$$

⇒ a- option.

Q-14 ⇒ $X \sim N(0, \sigma^2)$

let $Y = |X|$

$$\Rightarrow F_Y(y) = P[Y \leq y] = P[-y \leq X \leq y] = F_X(y) - F_X(-y)$$

$$\Rightarrow f_Y(y) = f_X(y) + f_X(-y) = 2 \cdot f_X(y).$$

$$\Rightarrow f_Y(y) = 2 \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2\sigma^2}(y-0)^2} = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\sigma} \cdot e^{-\frac{1}{2} y^2/\sigma^2} \quad ; \; y \geq 0.$$

⇒ c- option

Q-15 ⇒ First of all we define p.d.f of X as

$$f(x) = \left(1-\frac{1}{n}\right)\cdot N\left(0,\frac{1}{n}\right) + \frac{1}{n}\cdot N\left(1,\frac{1}{n}\right)$$

$$\Rightarrow E(x) = \left(1-\frac{1}{n}\right)\cdot E\left(N\left(0,\frac{1}{n}\right)\right) + \frac{1}{n}\cdot E\left(N\left(1,\frac{1}{n}\right)\right)$$

$$= \left(1-\frac{1}{n}\right)\cdot 0 + \frac{1}{n}\cdot 1 = \frac{1}{n} \Rightarrow \text{c-option}$$

Q-16 ⇒ $X \sim U(0,1)$

$$\Rightarrow E\left(\frac{1}{x}\right) = \int_0^1 \frac{1}{x}\,dx = \left[\log x\right]_0^1 = 0 - \log(0) = \infty$$

$\Rightarrow E\left(\frac{1}{x}\right)$ does not exist

$\Rightarrow V\left(\frac{1}{x}\right)$ does not exist ⇒ d-option

Q-17 ⇒ $f(x/\lambda) = \lambda\cdot e^{-\lambda x}$; $x>0$; $\lambda>0$.

$\lambda \sim \gamma(a, K)$

$$\Rightarrow f(x,\lambda) = f(\lambda)\cdot f(x/\lambda) = \frac{a^K\cdot e^{-a\lambda}\cdot \lambda^{K-1}}{\Gamma K}\cdot \lambda\cdot e^{-\lambda x}$$

$$\Rightarrow f(x) = \int_{\lambda=0}^{\infty} \frac{\lambda^K}{\Gamma K}\cdot a^K\cdot e^{-\lambda(a+x)}\,d\lambda$$

$$= \frac{a^K}{\Gamma K}\int_{\lambda=0}^{\infty} e^{-(a+x)\cdot\lambda}\cdot \lambda^{K+1-1}\,d\lambda = \frac{a^K}{\Gamma K}\cdot\frac{\Gamma K+1}{(a+x)^{K+1}}$$

$$\Rightarrow f(x) = \frac{K\cdot a^K}{(a+x)^{K+1}} \quad ; \quad x>0 \;:\; K>0.$$

⇒ b-option

Q-18 $\Rightarrow$ $E(X_i) = \mu$; $V(X_i) = \sigma^2$; $E(X_i - \mu)^4 = \sigma^4 + 1$

let $p = P\left[\sigma^2 - \frac{1}{\sqrt{n}} \le \frac{\sum (X_i - \mu)^2}{n} \le \sigma^2 + \frac{1}{\sqrt{n}}\right]$

$= P\left[-\frac{1}{\sqrt{n}} \le \frac{\sum (X_i - \mu)^2}{n} - \sigma^2 \le \frac{1}{\sqrt{n}}\right]$

$= P\left[-\frac{1}{\sqrt{n}} \le \frac{\sum [(X_i - \mu)^2 - \sigma^2]}{n} \le \frac{1}{\sqrt{n}}\right]$

$\Rightarrow p = P\left[-1 \le \frac{\sum [(X_i - \mu)^2 - \sigma^2]}{\sqrt{n}} \le 1\right]$ — (i)

let $Y_i = (X_i - \mu)^2 - \sigma^2 \Rightarrow E(Y_i) = E(X_i - \mu)^2 - \sigma^2 = \sigma^2 - \sigma^2 = 0$

and $V(Y_i) = V(X_i - \mu)^2 = E[X_i - \mu]^4 - [E(X_i - \mu)^2]^2$

$\Rightarrow V(Y_i) = \sigma^4 + 1 - \sigma^4 = 1$

let $S_n = \sum_{i=1}^{n} Y_i \Rightarrow E(S_n) = 0$ and $V(S_n) = n$

$\Rightarrow$ by C.L.T.

$$\frac{S_n - 0}{\sqrt{n}} \xrightarrow{L} N(0,1) \text{ as } n \to \infty$$

$\Rightarrow \lim_{n \to \infty} p = \lim_{n \to \infty} P\left[-1 \le \frac{S_n}{\sqrt{n}} \le 1\right] = P[-1 \le Z \le 1] = 2P[0 \le Z \le 1]$

$\Rightarrow$ b-option

Q-19 $\Rightarrow$ $f(x) = e^{-x}$; $F(x) = 1 - e^{-x}$.

let Range $= W = X_{(n)} - X_{(1)}$.

$\Rightarrow g(w) = n(n-1) \int_{-\infty}^{\infty} f(x) [F(x+w) - F(x)]^{n-2} \cdot f(x+w) \cdot dx$

$$\Rightarrow g(w) = n(n-1)\int_0^\infty e^{-x}\left[1-e^{-x-w}-(1-e^{-x})\right]^{n-2} \cdot e^{-(x+w)} \cdot dx$$

$$= n(n-1)\int_0^\infty e^{-x}\left[e^{-x}\right]^{n-2}\left[1-e^{-w}\right]^{n-2} \cdot e^{-x} \cdot e^{-w}\, dx$$

$$= n(n-1) \cdot e^{-w}\left(1-e^{-w}\right)^{n-2}\int_0^\infty e^{-nx}\, dx$$

$$= n(n-1) \cdot e^{-w}\left(1-e^{-w}\right)^{n-2}\left[\frac{e^{-nx}}{-n}\right]_0^\infty$$

$$= (n-1)e^{-w}\left(1-e^{-w}\right)^{n-2} \quad ; \quad 0<w<\infty$$

Here $n=5$, so

$$g(w) = 4e^{-w}\left(1-e^{-w}\right)^3 \quad ; \quad 0<w<\infty$$

$\Rightarrow$ b-option

Q-20 $\Rightarrow$ Here a, b, c are three Independent r.v's with Values $1, 2, 3, 4, 5, 6$. with Same prob^t $\frac{1}{6}$.

$$\Rightarrow E(a) = E(b) = E(c) = \frac{1}{6}[1+2+3+4+5+6] = \frac{7}{2}$$

and $E(a^2) = E(b^2) = E(c^2) = \frac{1}{6}[1^2+2^2+3^2+4^2+5^2+6^2] = \frac{91}{6}$

$$\Rightarrow V(a) = V(b) = V(c) = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$$

Since $X = a+b$ and $Y = a+c$

$\Rightarrow$ $E(X) = 7$ and $E(Y) = 7$

$V(X) = \frac{35}{6}$ and $V(Y) = 35/6$

Now $Cov(X,Y) = Cov(a+b,\ a+c) = V(a) = 35/12$

$\Rightarrow \ r(X,Y) = \dfrac{35/12}{35/6} = \dfrac{1}{2} = 0.5 \ \Rightarrow$ b-option

Q-21 $\Rightarrow$ $E(X) = E(Y) = \dfrac{N+1}{2}$

$V(X) = V(Y) = \dfrac{N^2-1}{12}$

$\Rightarrow E(X-Y)^2 = E[(X-E(X)) - (Y-E(Y))]^2$ $\{E(X) = E(Y)$

$= E[X-E(X)]^2 + E[Y-E(Y)]^2 - 2E[(X-E(X))(Y-E(Y))]$

$\Rightarrow E(X-Y)^2 = \sigma_x^2 + \sigma_y^2 - 2r(X,Y)\cdot\sigma_x\cdot\sigma_y$

$\Rightarrow E(X-Y)^2 = \sigma_x^2[1+1-2r(X,Y)]$

$\Rightarrow 1 - r(X,Y) = \dfrac{E(X-Y)^2}{2\cdot\sigma_x^2} = \dfrac{6E(X-Y)^2}{N^2-1}$

$\Rightarrow r(X,Y) = 1 - \dfrac{6E(X-Y)^2}{N^2-1} \ \Rightarrow$ a-option

Q-22 $\Rightarrow$ $M(t_1,t_2) = [a(e^{t_1+t_2}+1) + b(e^{t_1}+e^{t_2})]^2$

$\Rightarrow M_{X_1}(t) = M(t,0) = [a(e^t+1) + b(e^t+1)]^2 = \frac{1}{4}(e^t+1)^2$

$\Rightarrow E(X_1) = \left[\frac{1}{4}\cdot 2(e^t+1)e^t\right]_{t=0} = 1$

and $E(X_1^2) = \left[\dfrac{e^t(e^t) + e^t(e^t+1)}{2}\right]_{t=0} = \dfrac{3}{2}$

$\Rightarrow V(X_1) = V(X_2) = \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow V(X_1) + V(X_2) = 1 \ \Rightarrow$ a-option

Q-23 ⇒ $f_Y(y) = \int_{x=0}^{\infty} \frac{1}{2} \cdot y \cdot e^{-xy} dx = \frac{y}{2}\left[\frac{e^{-xy}}{-y}\right]_0^{\infty} = \frac{1}{2}$; $0 < y < 2$

$\Rightarrow f(x/y) = \dfrac{\frac{1}{2} y \cdot e^{-xy}}{\frac{1}{2}} = y \cdot e^{-xy}$; $0 < x < \infty$; $0 < y < 2$

$\Rightarrow E[X/_{Y=y}] = \int_{x=0}^{\infty} x \cdot y e^{-xy} dx = y \int_0^{\infty} e^{-yx} \cdot x^{2-1} dx = \frac{1}{y}$

⇒ Regression curve of X on Y is
$X = E[X/_{Y=y}] \Rightarrow x = \frac{1}{y} \Rightarrow x \cdot y = 1 \Rightarrow$ b-option

Q-24 ⇒ $X_i \overset{iid}{\sim} N(0,1) \Rightarrow \Sigma X_i \sim N(0, n) \Rightarrow \frac{\Sigma X_i}{\sqrt{n}} \sim N(0,1)$.

Since $\bar{X}$ and $\Sigma(X_i - \bar{X})^2$ are Independent and
$\Sigma(X_i - \bar{X})^2 \sim \chi^2_{n-1}$

$$\Rightarrow \frac{\frac{\Sigma X_i}{\sqrt{n}}}{\sqrt{\Sigma(X_i - \bar{X})^2 / n-1}} \sim t_{n-1} \Rightarrow \sqrt{\frac{n-1}{n}} \cdot U \sim t_{n-1}$$

$\Rightarrow E\left[\sqrt{\frac{n-1}{n}} \cdot U\right] = 0 \Rightarrow E(U) = 0 \Rightarrow$ b- option

Q-25 ⇒ This is the case of trinomial distribution

$(X, Y, n-X-Y) \sim m.N(3, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, n)$; $n = 900$.

$\Rightarrow Cov(X,Y) = -np_1p_2 = -100. \Rightarrow$ c-option

Q-26 ⇒ Non-parametric methods are based on Mild Assumptions ⇒ a-option

Q-27 ⇒ The maximum possible number of Runs are 10 because, symbols of Two types are equal in number ⇒ c-option

Q-28 ⇒ Rank Correlation is defined as

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \Rightarrow \text{a-option}$$

Q-29 ⇒ Wald-Wolfowitz Run Test for Two Samples is affected when the Ties occur Between Samples. ⇒ b-option

Q-30 ⇒ let us define some Events as

$A_1 \rightarrow$ Coin is fair

$A_2 \rightarrow$ Coin is biased

$A \rightarrow$ geting Head.

$$\Rightarrow P(A/A_1) = \frac{1}{2} \quad \text{and} \quad P(A/A_2) = 2/3$$

$$P(A_1) = \frac{m}{N} \quad \text{and} \quad P(A_2) = \frac{N-m}{N}$$

$$\Rightarrow P[A_1/A] = \frac{P(A/A_1)\cdot P(A_1)}{P(A/A_1)\cdot P(A_1) + P(A/A_2)\cdot P(A_2)} = \frac{3m}{4N-m} \Rightarrow \text{d-option}$$

Q-31 ⇒ $x, y \in W = \{0, 1, 2, 3, \cdots\}$

it is given that $x+y = 200$.

we know that Max of $x \cdot y$ is possible only when $x = y = 100$.

⇒ Required prob^t is $= P\left[x \cdot y \geq \frac{3}{4} \cdot 100 \times 100\right]$

⇒ $P[x \cdot (200-x) \geq 7500] = P[50 \leq x \leq 150]$

$= \frac{101}{201} = \frac{\text{favourable cases}}{\text{Total cases}}$

⇒ b-option

Q-32 ⇒ The prob^t of A winning $= P(A) + P(\bar{A}\bar{B}A) + \cdots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots = \frac{2}{3}$

⇒ prob^t of B winning $= 1 - \frac{2}{3} = \frac{1}{3}$

⇒ The Expectation of geting A $= 30 \times \frac{2}{3} = 20$ Rs

and " " " " B $= 30 \times \frac{1}{3} = 10$ Rs

⇒ c-option

Q-33 ⇒ let E_1, E_2, E_3, E_4 be the Events that the bag contains 1 white, 2 white, 3 white and 4 white balls.

⇒ $P(E_J) = \frac{1}{4} \; \forall \; J = 1, 2, 3, 4.$

Let $W \rightarrow$ Event that the ball drawn is white.

$\Rightarrow P(E_4/W) = p/15$

$$\Rightarrow \frac{P(E_4) \cdot P(W/E_4)}{\sum_{J=1}^{4} P(E_J) \cdot P(W/E_J)} = \frac{p}{15} \Rightarrow \frac{\frac{1}{4} \cdot 1}{5/8} = \frac{p}{15} \Rightarrow p = 6$$

$\Rightarrow$ a- option

Q-34 $\Rightarrow P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \dfrac{1 - P(A \cup B)}{P(\bar{B})} \Rightarrow$ c- option

Q-35 $\Rightarrow \gamma_1 = +\sqrt{\beta_1} = \sqrt{\mu_3^2/\mu_2^3}$ and $\beta_2 = \mu_4/\mu_2^2$

$\Rightarrow 1 = \dfrac{\mu_3^2}{16^3}$ and $4 = \dfrac{\mu_4}{16^2}$

$\Rightarrow \mu_3 = 64$ and $\mu_4 = 1024$

$\Rightarrow$ b - option

Q-36 $\Rightarrow P[X = x] = (2x+1)a \quad ; \quad x = 0, 1, \ldots 8.$

$\Rightarrow a \sum_{x=0}^{8} (2x+1) = 1 \Rightarrow a[1+3+5+7+9+11+13+15+17] = 1$

$\Rightarrow a = \dfrac{1}{81} \Rightarrow$ a- option

Q-37 $\Rightarrow M_X(t) = \frac{2}{5} + \frac{1}{3}e^{2t} + \frac{4}{15}e^{3t}.$

$E(X) = \left[0 + \frac{2}{3}e^{2t} + \frac{12}{15}e^{3t}\right]_{t=0} = \dfrac{22}{15} \Rightarrow$ d- option

Q-38 ⇒ I, II, and III are true by the property of c.d.f. ⇒ b-option

Q-39 ⇒ According to the given c.d.f

$X \sim U(0,1)$ with p.d.f as $f(x) = \begin{cases} 1 & ; 0 \le x \le 1 \\ 0 & ; \text{otherwise} \end{cases}$

⇒ S: is true

Since $F(x)$ is Continuous $\forall x$

⇒ P: is False.

⇒ b-option

Q-40 ⇒ let X denote the marks, He get for attained one question.

⇒ | | | |
|---|---|---|
| X: | 1 | -0.25 |
| $P(x)$: | $\frac{1}{4}$ | $\frac{3}{4}$ |

$$E(x) = 1 \cdot \frac{1}{4} - 0.25 \times \frac{3}{4} = 0.0625$$

Hence Expected Value of marks for attend 80 question $= 80 \times 0.0625 = 5$

⇒ c-option

Q-41 ⇒ X and Y are Independent ⇒ $E(X \cdot Y) = E(X) \cdot E(Y)$
$\not\Leftarrow$

⇒ d-option

Q-42 ⇒ Here X may be continuous or discrete r.v so

$$P[2 < X \leq 5] = F(5) - F(2)$$

⇒ c-option

Q-43 ⇒ $X \sim b(n, p) \Rightarrow V(X) = npq$

Now $V\left(\frac{X}{n}\right) = \frac{1}{n^2} V(X) = \frac{pq}{n} = \frac{p(1-p)}{n} \Rightarrow$ a-option

Q-44 ⇒ $\left.\begin{array}{l} X \sim \gamma(m) \\ Y \sim \gamma(n) \end{array}\right\}$ Independent

$\Rightarrow \frac{X}{X+Y} \sim \beta_1(m, n) \Rightarrow$ b-option

Q-45 ⇒ $E(x^4) = \int_0^\infty x^4 \cdot \frac{1}{2} e^{-x/2} dx = \frac{1}{2} \int_0^\infty e^{-x/2} \cdot x^{5-1} dx$

$\Rightarrow E(x^4) = \frac{1}{2} \cdot \frac{\sqrt{5}}{\left(\frac{1}{2}\right)^5} = 384 \Rightarrow$ c-option

Q-46 ⇒ Continuous Rectangular Distribution

⇒ c-option

Q-47 ⇒ $X \sim b\left(5, \frac{1}{2}\right)$ and $Y \sim U(0,1)$.

$$\frac{P[X+Y \leq 2]}{P[X+Y \geq 5]} = \frac{P[X=0 \,;\, 0 \leq Y \leq 1] + P[X=1 \,;\, 0 \leq Y \leq 1] + P[X=2, Y=0]}{1 - P[X+Y < 5]}$$

$$= \frac{P(X=0) \cdot 1 + P(X=1) \cdot 1 + P(X=2) \cdot 0}{1 - P(X=0) \cdot 1 - P(X=1) \cdot 1 - P(X=2) \cdot 1 - P(X=3) \cdot 1 - P(X=4) \cdot 1}$$

$$= \frac{\left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5}{1-\left(\frac{1}{2}\right)^5 - 5\left(\frac{1}{2}\right)^5 - 10\left(\frac{1}{2}\right)^5 - 10\left(\frac{1}{2}\right)^5 - 5\left(\frac{1}{2}\right)^5} = 6$$

$\Rightarrow$ b-option

Q-48 $\Rightarrow$ $\left.\begin{array}{l} X \sim P(1) \\ Y \sim P(2) \end{array}\right\}$ Independent $\Rightarrow X+Y \sim P(3)$

$\Rightarrow P[0 < X+Y < 3] = P[X+Y = 1, 2]$

$$= \frac{e^{-3} \cdot 3^1}{1!} + \frac{e^{-3} \cdot 3^2}{2!} = 7.5\, e^{-3}$$ $\Rightarrow$ d-option

Q-49 $\Rightarrow$ $X_i \sim N(0,1) \Rightarrow E|X_i| = \sqrt{\frac{2}{\pi}}$

$$\Rightarrow \lim_{n \to \infty} \frac{1}{2n} \sum_{i=1}^{n} E|X_i| = \lim_{n \to \infty} \frac{1}{2n} \cdot n \cdot \sqrt{\frac{2}{\pi}} = \frac{1}{\sqrt{2\pi}}$$

$\Rightarrow$ d-option

Q-50 $\Rightarrow$ $P(X=x) = \left(\frac{1}{2}\right)^x \; ; \; x = 1, 2, 3, \ldots$

Let m be the median

$$\Rightarrow \sum_{x=1}^{m} p(x) = \frac{1}{2} \Rightarrow \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^m = \frac{1}{2}$$

$$\Rightarrow \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^m\right]}{1-\frac{1}{2}} = \frac{1}{2} \Rightarrow \left(\frac{1}{2}\right)^m = \frac{1}{2} \Rightarrow m = 1$$

$\Rightarrow$ a-option

Q-51 ⇒ b - option

Q-52 ⇒ b - option

Q-53 ⇒ c - option

Q-54 ⇒ c - option

Q-55 ⇒ a - option

Q-56 ⇒ d - option

Q-57 ⇒ c - option

Q-58 ⇒ c - option

Q-59 ⇒ b - option

Q-60 ⇒ a - option

Q-61 ⇒ d - option

Q-62 ⇒ c - option

Q-63 ⇒ b - option

Q-64 ⇒ b - option

Q-65 ⇒ c - option

Q-66 ⇒ c - option

Q-67 ⇒ d - option

Q-68 ⇒ d - option

Q-69 $\Rightarrow$

$$(10001010)_2 = 0\times2^0 + 1\times2^1 + 0\times2^2 + 1\times2^3 + 0\times2^4 + 0\times2^5 + 0\times2^6 + 1\times2^7$$

$$= 2 + 8 + 128$$

$$= (138)_{10}$$

$\Rightarrow$ a- option

Q-70 $\Rightarrow$ $(125)_8 = 5\times8^0 + 2\times8^1 + 1\times8^2$

$$= 5 + 16 + 64 = (85)_{10}$$

$\Rightarrow$ b- option

Ans 11

$$\int_0^1 (x^3 - cx^2)\,dx$$

Question say's, it is exact by approximating the integral from Trapezoidal Rule.

$$\text{let } I = \int_0^1 (x^3 - cx^2)\,dx = \left[\frac{x^4}{4} - c\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{4} - \frac{c}{3}$$

Trapezoidal Rule.

$$T = \int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}\Big[(\underset{1^{st}\text{ term}}{y_0} + \underset{\text{last term}}{y_n}) + 2(\underbrace{y_1 + y_2 + \cdots + y_{n-1}}_{\text{Remaining terms}})\Big]$$

where $n \in \mathbb{N}$.

$h = 1$

0 —— 1

So, here

x	$f(x) = x^3 - cx^2$
0	0
1	$1 - c$

$$T = \frac{1}{2}[0 + 1 - c] = \frac{(1-c)}{2}$$

$$I = T$$

$$\Rightarrow \frac{1}{4} - \frac{c}{3} = \frac{1}{2} - \frac{c}{2}$$

$$\Rightarrow c = 1.5$$

Ans 2.

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
x_0	10	y_0 1754	894	22
x_1	15	y_1 2648	916	
x_2	20	y_2 3564		

Given : $y = 3000$

By using Inverse Interpolation

$$x^{(1)} = \frac{y - y_0}{\Delta y_0} = \frac{3000 - 1754}{894} \simeq 1.39$$

Put $x = (1.39)$

$$x^{(2)} = \frac{y - y_0}{\Delta y_0} - \frac{x(x-1)}{2!}\frac{\Delta^2 y_0}{\Delta y_0}$$

$$= 1.39 - \frac{x^{(1)}(x^{(1)} - 1)}{2!}\frac{\Delta^2 y_0}{\Delta y_0}$$

$h = 5$

$$= 1.39 - \frac{1.39(1.39 - 1)}{2!} \times \frac{22}{894}$$

$$= 1.3792$$

Required Value of $x = x_0 + ph$

$$= 10 + (1.3792) \times 5$$

$$= 16.896$$

Ans 3

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$	$\Delta^4 u_x$	$\Delta^5 u_x$
0	3	9	60	-10	-259	755
1	12	69	50	-269	496	
2	81	119	-219	227		
3	200	-100	8			
4	100	-92				
5	8					

Hence $\Delta^5 u_0 = 755.$

Ans 74

Euler's Method

$$\frac{dy}{dx} = f(x,y) = x^2 + y^2 \quad \text{—(1)}$$

$y(x_0) = y_0$ i.e $y(0) = 1$ (in this ques)

n^{th} approximation sol of (1) is

$$y_n = y_{n-1} + h\, f(x_{n-1}, y_{n-1})$$

where $x_{n-1} = x_0 + (n-1)h$

$h = 0.1$ (given)

$x_0 = 0$

$x_1 = x_0 + h = 0 + 0.1 = 0.1$

$x_2 = x_0 + 2h = 0 + 2(0.1) = 0.2$

So, you have to calculate till y_2.

$$y_1 = y_0 + h\, f(x_0, y_0)$$

$$= 1 + 0.1(x_0^2 + y_0^2)$$

$$= 1 + (0.1)(0^2 + 1^2)$$

$$= 1 + 0.1 = 1.1$$

$$y_2 = y_1 + h\, f(x_1, y_1)$$

$x_1 = x_0 + h = 0.1$

$$= 1.1 + (0.1)\left((0.1)^2 + (1.1)^2\right)$$

$$= 1.1 + (0.1)(0.01 + 1.21)$$

$$= 1.222.$$

Ans 75. Same as 73. (Thanks)

Ans 76 $[\Delta^2 x^5]_{x=0}$; $h = 1$

Use differences of zero.

$$[\Delta^n x^m]_{x=0} = n^m - {}^nc_1 (n-1)^m + {}^nc_2 (n-2)^m \cdots + {}^nc_{n-1} (-1)^{n-1}$$

$$[\Delta^2 x^5]_{x=0} = \Delta^2 0^5 = 2^5 - {}^2c_1 (2-1)^5$$

$$= 32 - 2$$

$$= 30$$

Ans 77 We have to tell "NOT TRUE" statement

so option (a) is not true

hence Divided differences are symmetric function in general.

Ans 78 (a) is correct i.e (b) is incorrect

$$\therefore \Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{\delta^2}{4}}$$

(c) is incorrect, bcz, $E = 1 + \Delta$

i.e $\Delta \equiv E - 1$

$\rightarrow$ we can easily prove by putting $\delta = E^{1/2} - E^{-1/2}$ in R.H.S of (a).

TEST SERIES PAPER-1 (TEST-1) SOLUTION

Ans 79

$$\frac{dy}{dt} = f(t) \; ; \; y(0) = 0$$

R-K Method $\Rightarrow \frac{dy}{dx} = f(x, y)$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Result

where $k_1 = h f(x_0, y_0)$; $k_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$; $k_4 = h f(x_0 + h, y_0 + k_3)$

$\frac{dy}{dt} = f(t)$ (single variable only). $y(0) = 0$

i.e $x_0 = 0$ & $y_0 = 0$

$$y_1 = 0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = h f(0)$ $k_2 = h f(\frac{h}{2})$

$k_3 = h f(\frac{h}{2})$; $k_4 = h f(h)$

$$y_1 = \frac{1}{6}(h f(0) + 2h f(\frac{h}{2}) + 2h f(\frac{h}{2}) + h f(h))$$

$$y_1 = \frac{h}{6}(f(0) + 2^2 f(\frac{h}{2}) + f(h)).$$

Ans 80

$$\text{Area} = \int_{7.47}^{7.52} f(x)\,dx$$; $h = 0.01$ (from the given data)

Apply Trapezoidal Rule $= \frac{h}{2}((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}))$

$$= \frac{0.01}{2}[(1.93 + 2.06) + 2(1.95 + 1.98 + 2.01 + 2.03)]$$

$$= 0.09965$$

ISS COACHING

Guided by - Sudhir Sir 9999001310

Sudhirdse1@gmail.com

www.isscoaching.com

2513, Basement, Hudson Lane Beside HDFC Bank Opp. Laxmi Dairy, GTB Nagar New Delhi: 110009